Physics 200A **Theoretical Mechanics** **Fall 2016**

Problem Set II: Due TBA

> "I do not like to be dunned and teased by foreigners about mathematical things."
> – Isaac Newton

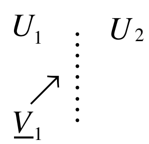
1.) Consider the nonlinear string problem, as developed in class.

a.) Derive the Lagrangian and Lagrangian equations of motion. What is the pragmatic criterion for the reduction of these to a linear wave equation?

b.) Derive the string Hamiltonian and the Hamiltonian equations of motion.

c.) Derive an energy theorem for the linear string. Discuss its correspondence to the Poynting theorem. What are the wave energy density and energy flux density?

2.) Consider a particle moving in:



a.) V_1 is the initial velocity. How does the direction change? What is V_2?

b.) Find the ratio of times in the same path for particles with different masses but the same U.

c.) Find the ratio of times in the same path for particles with the same mass but moving different potentials U_1, U_2, where $U_2/U_1 = k$, a constant.

d.) What problem in optics does this problem resemble?

3.) FW: 6.4

4.) FW: 6.2

5.) FW: 6.3 (I assume you are familiar with the heavy symmetric top from undergrad mechanics. If not, please read up on it.)

6.) For a particle of mass m moving thru a potential $U(x)$, the time-independent Schrodinger equation is:

$$-\left(\frac{\hbar^2}{2m}\right)\frac{d^2\Psi}{dx^2}+U\Psi=E\Psi\ .$$

$\Psi(x)$ is the wave function and E is the energy. Ψ^{*} is the complex conjugate to Ψ .

Show that the Schrodinger equation is the Lagrange Equation for:

$$L=\frac{-\hbar^2}{2m}\left|\frac{d\Psi}{\partial x}\right|^2-\Psi^*(U-E)\Psi\ .$$

7.) Consider the vibrational motion of a system with configuration coordinate q and Hamiltonian

$$H(q,p)=p^2/2m+V(q)$$

vibrating between the limits q_1 and q_2 at energy *E*. Show that the period can be written in the form

$$T(E)=2\sqrt{2m}\frac{d}{dE}\int_{q_1}^{q_2}dq[E-V(q)]^{\frac{1}{2}}\ .$$

Hence show that if

$$V(q)=V_0(q)+\in V_1(q)\ ,$$

then for small $\in$ the period can be written in the approximate form

$$T(E)=T_0(E)+\in T_1(E)$$

with

$$T_1(E)=-\sqrt{2m}\frac{d}{dE}\int_{q_1^0}^{q_2^0}dq\frac{V_1(q)}{[E-V_0(q)]^{\frac{1}{2}}}$$

where q_1^0 and q_2^0 are the limits of the motion in $V_0(q)$ with energy *E*.
Take particular care with these limits.